Analysis Anomalous Doppler Effect from quantum theory to classical dynamic simulation

Abstract:

A quantum model combined with angular momentum conservation is established to analyze the process of Normal doppler effect and Anomalous Doppler Effect, illustrating that the resonance process is related to the angular momentum of the wave. The angular momentum resonant condition is numerically tested, and the energy change ratio between parallel and gyrokinetic energies during electron–wave resonance is calculated, showing strong agreement with quantum theory.

I. Introduction:

The Anomalous Doppler Effect (ADE)[[1-4](#_ENREF_1)], in which the observed frequency shift behaves contrary to the conventional Doppler Effect under specific conditions, was first theoretically predicted by Soviet physicist Vitaly L. Ginzburg[[5](#_ENREF_5)]. This phenomenon occurs when a moving system’s velocity exceeds the phase velocity of light in the medium, it transfers its kinetic energy to its internal energy while emitting radiation. A notable example, discussed by Frank in his 1958 Nobel lecture[[2](#_ENREF_2)], demonstrates that radiation emission does not result from atomic transitions from a higher (excited) state to a lower state, as is typical, but rather occurs inversely—from a lower state to a higher state—where the energy is supplied by the system’s translational kinetic energy. This intriguing theoretical prediction has attracted significant attention and has motivated extensive research[[6-14](#_ENREF_6)].

In 1967, Artsimovich[[15](#_ENREF_15)] observed discrepancies in tokamak experiments: measurements of electron temperature derived from diamagnetic signals stronger than derived from electrical conductivity measurement. This anomaly, unrecognized at the time, may represent the first experimental observation of ADE. It was not until 1968 that B. B. Kadomtsev[[16](#_ENREF_16)]  identified the that cause as ADE, wherein electron’s longitudinal velocity scatter to transverse velocity under resonant ADE conditions. This process amplifies the diamagnetic effect beyond contributions from thermal motion alone. After that, more phenomena related to ADE are observed, such as electron beam scattering in magnetic field vacuum tube[[4](#_ENREF_4)], wave radiation [[17-19](#_ENREF_17)] and runaway electron instability in tokamaks [[20](#_ENREF_20), [21](#_ENREF_21)]. Applications based on ADE have also emerged in various areas, such as high-power microwave generation [[22](#_ENREF_22)] and runaway electron suppression in tokamaks[[11](#_ENREF_11), [23](#_ENREF_23)].

The physics of the Anomalous Doppler Effect (ADE) was previously explained based on the quantum analysis provided by Frank and Ginzburg[[2](#_ENREF_2), [24](#_ENREF_24)]. In this paper, building on Ginzburg’s quantum analysis and incorporating angular momentum conservation, we present a more detailed analysis of ADE, offering further insights into the relationship between ADE and wave angular momentum. Despite the simplicity of the model, to the best of our knowledge, the analysis of angular momentum conservation during the ADE process has not been presented or mentioned before.

Additionally, a numerical simulation of a single particle resonating with an electromagnetic (EM) wave in the presence of uniform static electric and magnetic fields is conducted based on classical dynamical equations. This simulation demonstrates the relationship between the wave’s angular momentum and the resonance condition. The energy transfer ratio from the electron's kinetic energy to its gyrokinetic energy during the resonance with the EM wave is computed numerically, and the results show strong agreement with the formula derived from quantum analysis.

The remainder of this paper is organized as follows. Section II presents the quantum analysis based on angular momentum conservation. Section III describes the numerical setup and methodology, including illustrations of the temporal evolution of velocity and kinetic energy. Section IV discusses the energy transfer ratio and polarization characteristics. Finally, Section V provides a brief discussion and conclusion.

II. Quantum analysis of ADE

We begin with the quantum theory proposed by V.L. Ginzburg[[25](#_ENREF_25)] based on the energy-momentum conservation, and then incorporate angular momentum conservation to illustrate the relationship of wave’s angular momentum and Landau level n under the resonant condition , where is wave’s angular frequency , represents the wave vector, means the velocity of electron and is the electron cyclotron angular frequency in static magnetic field, n = is the quantum number of the Landau levels[[26](#_ENREF_26)].

When a charged particle moves through a medium at a speed greater than the phase velocity of light in that medium, it induces polarization in the surrounding molecules. As these molecules return to their equilibrium state, they emit electromagnetic radiation. The constructive interference of these emissions produces the characteristic Cherenkov radiation, forming a cone-shaped wavefront as shown in fig.1. The direction of Cherenkov radiation is constrained to the Cherenkov radiation angle ,where c′ is the speed of light in the medium and v is the velocity of the charged particles.

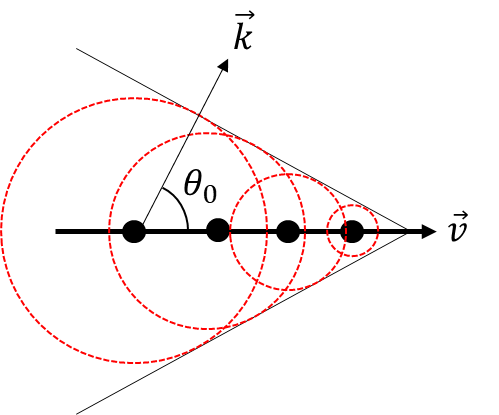


Figure . Schematic diagram of Cherenkov Radiation. The black points stand for the snapshot of the electron at different times, the read dash circle refers to the current radiation surface from the previous electron.

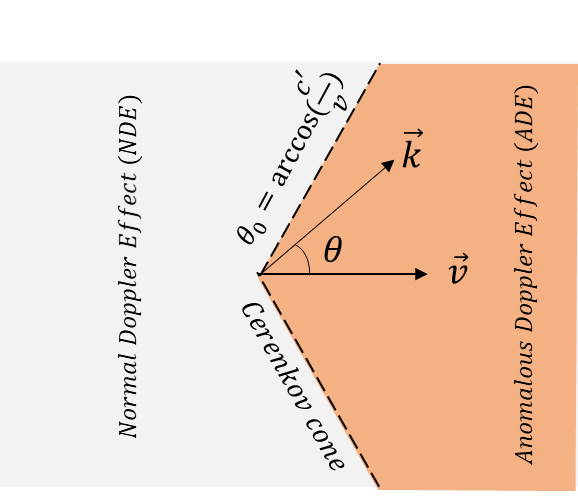


Figure .The region of Anomalous Doppler Effect (ADE) and Normal Doppler Effect (NDE).

However, when the electron is replaced by a system possessing internal energy—such as an oscillator or a cyclotron electron in a magnetic field—the direction of the emitted photon is no longer determined by the interference of secondary waves and can instead occur in any direction. Considering a scenario where the system emits a photon with angular frequency 𝜔 and wavevector k, the emission process must satisfy both energy and momentum conservation:

Here the and represent the kinetic energy and internal energy of the system while subscripts of 1 and 2 refer to before and after emitting a photon. p represents the momentum of the system and ℏ represnts reduced Planck's constant. Assumpting that photon’s energy is far less than the initial kinetic energy , the losses of kinetic energy after emitting a photon can be expressed as , where v is the velocity of the system before emitting a photon and **.** Thus, the change of internal energy can be expressed as

Here, . When the system's velocity exceeds the speed of light in the medium , the sign of allows the radiation to be categorized into three distinct regions, as illustrated in fig. 2.

* For , . The system produces photons by consuming its own internal and kinetic energy, this region refers to the Normal Doppler Effect (NDE).
* For , , the loss of kinetic energy by the system is completely converted into photon energy; this line refers to the Cerenkov Effect.
* For , , this region is referred to the Anomalous Doppler Effect (ADE), where the system gains internal energy after emitting photons. It means the loss of kinetic energy is converted to photons and the system’s internal energy.

In previous paper, the change of internal energy is given as where m = represent the Landau level ,as given by V.L. Ginzburg[[25](#_ENREF_25)] ,Coppi[[26](#_ENREF_26)],Frolov[[27](#_ENREF_27)] ,Frank[[2](#_ENREF_2)],Tamm[[1](#_ENREF_1)] and Nezlin[[6](#_ENREF_6)] . The above content revisits the foundational work of V.L. Ginzburg[[25](#_ENREF_25)]. In the present paper, it is further demonstrated that m actually represents the quantum number associated with the angular momentum of the emitted photon.

Let’s consider the process in which an electron cyclotron system under a uniform magnetic field emits a photon, as shown in fig.3. The moving electron has the velocity vz along the background magnetic field and the cyclotron velocity. The kinetic energy along z is , where refers to the Lorentz factor. The internal energy represents as . Assume the angular moment of the system before and after emitting a photon is L1 and L2, respectively. The angular moment of photon is n. According to the angular momentum conservation, we have

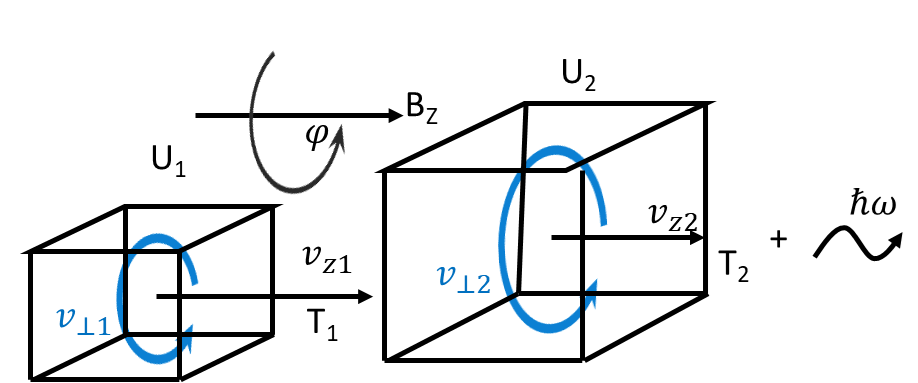


Figure . Schematic diagram of electron cyclotron system before and after emitting a photon. Here U2>U1, T2<T1.

Since the magnetic field is aligned along z direction, the angular momentum of electron along z is represented as Lz. According to the quantum theory, the electron wave in the static magnetic field can be expressed as

With the term represents as normalized coefficient, A is the vector potential and s is the position. For gyro-motion electron in magnetic field, s = rφ , where r refers to cyclotron radius and φ refers to cyclotron angle. The z component of the orbital angular momentum operator can be expressed in spherical coordinates as

Combining eq. () with eq. (), we have

As a result, the eigenvalue of can be expressed as

With ,and , the eq. () is presented as

Here is the electron rest mass, is the Lorentz factor and is the electron cyclotron frequency in rest frame (). The angular momentum conservation in z direction is The variation in the angular momentum of the electron along z is presented as

Here, m is the quantum number of photon’s angular momentum in the z direction. The internal energy change is given by , with the eq. (), it can be transformed as :

According to the eq. and eq., the change in electron energy could be presented as

Which is same as the previous results [[1](#_ENREF_1), [2](#_ENREF_2), [6](#_ENREF_6), [26-28](#_ENREF_26)].Here, represents the loss of kinetic energy , represents the energy of the photon, and represents the change in the electron gro-kinetic energy (the internal energy change). The change ratio of internal energy and kinetic energy can be expressed as

This results is a critical criterion to compare with the classical dynamic simulation in the section 2. It is also proved based on classical theory in the Appendix.

After simpifying the eq., we finally have the classical wave-particle resonant condition

The variable m represents the quantum number associated with the angular momentum of the photon. Since a photon possesses both orbital angular momentum (and intrinsic spin angular momentum , where s = )[[29](#_ENREF_29)] , the total angular momentum can be expressed as . If we consider the plane wave, only the spin angular momentum will be included in this context (=0), there are two possible scenarios regarding the sign of m.

* For , where , the internal energy of the cyclotron electron decreases after emitting a photon. If the angular momentum quantum number m =1, the emitted photon exhibits right-hand circular polarization. This process is known as the NDE.
* For , , the cyclotron electron gains internal energy after emitting a photon. The emission photo will have left-hand circular polarization if the angular momentum quantum number m = -1. This process is known as the ADE.( The difference in the definition of left- and right-hand polarization for m in the paper (The angular momentum of photons in a circularly polarized beam) arises because is chosen, where m>0 corresponds to the same rotation sense as the electron's right-hand polarization)

While ADE and NDE describe spontaneous emission phenomena that occur without external field intervention, in our simulation model an external electromagnetic (EM) waves is introduced as resonant fields interacting with electrons in static magnetic and electric fields. This approach provides a framework for analysing ADE under resonant conditions, referred to here as Anomalous Doppler Resonance (ADR). Under such resonance, both emission and absorption processes can occur, depending on the phase relationship between the electron’s perpendicular velocity and the electric field of the E.M wave. A detailed analysis is provided in the Appendix.

Depsite nonlinear analyses of electron interactions with E.M waves—excluding static electric fields—have been presented in numerous studies[[30-38](#_ENREF_30)], fewer investigations have considered the influence of a static electric field during resonance with E.M waves. Due to the complexity of the nonlinear processes involved, analytical solutions are nearly impossible to obtain, making numerical simulations essential in this context.

For a external electromagnetic wave as plane wave, the wave angular moment number can be devided into . While for , it indicates that the resonant wave possesses a helicon structure. In this study, we consider only the plane wave case, with the ADE resonantce condition is given by , corresponding to left-circularly polarized wave and the NDE resonance condition given by correspond to right-circularly polarized wave.

Section 2 : Classical dynamic simulation of ADR

The ADE process has been analyzed based on quantum theory, demonstrating that the angular momentum of the emitting photon determines the resonance condition. Specifically, only angular momentum with m < 0 corresponds to the ADE process, while m >0 corresponds to the NDE process. These characteristics will be tested through the interaction of E.M wave and the electron during ADR and Normal Doppler Resonance (NDR), and the energy transfer ratio can also be verified through numerical simulations.

Section 2.1 : Numerical simulation setup

To analyse the resonant process from the perspective of classical dynamics and to provide a direct comparison between quantum and classical dynamic results, the following scenario is established: The uniform magnetic field is set along the z-direction. The electrostatic field , which on the opposite direction to as shown in fig. 4, is used to accelerate the electron. We will consider the interaction between a electron entering the system with velocity vz parallel to the magnetic field B0=BZ, and a linearly/ circularly polarized TEM wave, propagating in a homogeneous dielectric with index of refraction n > 1.

The induced linear polarized wave along can be sperated as the combined of right-circularly polarized wave (m = 1) and left-circularly polarized (m = -1) wave as , where . If the wavevector align in y-z plane with a crossing angle between z axis, the new coordinate unit vector in wave expression should be

The magnetic field of E.M wave is

The six-dimensional phase space of an electron, described by its position **r** and momentum **p,** are presented in eq.. The vectors **E** and **B** represent the total field, including both static and electromagnetic components. Here, c denotes the speed of light in vacuum, e represents to the electron’s charge and m0 is the electron’s mass in rest frame.

To simulate the evolution of r and p , the eq. is discreted as the form of eq., based on the Volume-Preserving Algorithm[[9](#_ENREF_9), [10](#_ENREF_10), [39](#_ENREF_39)]. Here the k is the iteration step and the operator Cay(A) denotes the Cayley transform of matrix A [[39](#_ENREF_39)].

The dimensionless parameters are momentum , magnetic field total electric field ,time step , and position respectively, where the is the electron cyclotron period () and is Lorentz factor. The dimensionless magnetic matrix B\* [[39](#_ENREF_39)]is writen as eq.

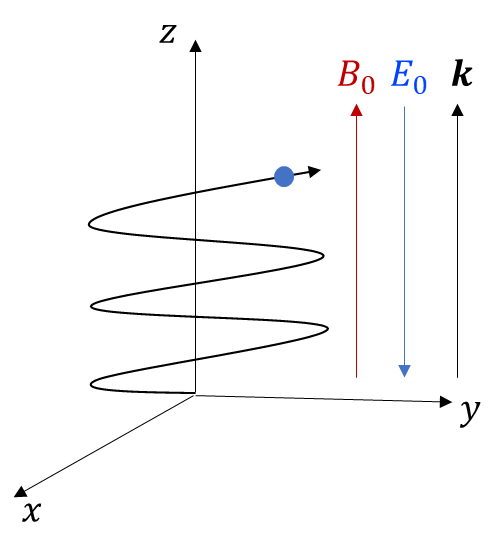


Figure .The uniform static magnetic field is set along the z axis, the electrostatic field E0 is oriented opposite to the B0 field, and the wavevector k is aligned parallel to the B0 field.

To illustrate the system evolution, the parameters are set as following: background magnetic field , wave angular frequency where , wavevector , the electric field component of the electromagnetic wave . The propagation of induced wave with linear polarization is parallel to z axis, and the electrostatic field is . The time resolution is always chosen to satisfy 50()) to ensure the accuracy of the simulation.

The evolution of the electron’s motion is shown in fig. 5. As the electron accelerates from stationary in the electrostatic field (fig. 5(b)), the resonant frequencies increase simultaneously (fig. 5 (a)). The change of parallel velocity caused by electromagnetic wave can be quantified as as shown in fig. 5 (c), where vz represents the parallel velocity under the given scenario, while vzE0​ denotes the parallel velocity resulting solely from the electrostatic field, which can be calculated using a theoretical equation as

The cyclotron velocity is shown in fig. 5 (d). The work done by electromagnetic wave is shown in fig. 5 (e), which can be calculated by integrating the power with time as ,and . Since all discrete date points are available from the simulation, it is no difficult to integrate all the discreate date over time. Fig. 5 (f) shows the gro-kinetic energy evolution with time, where .

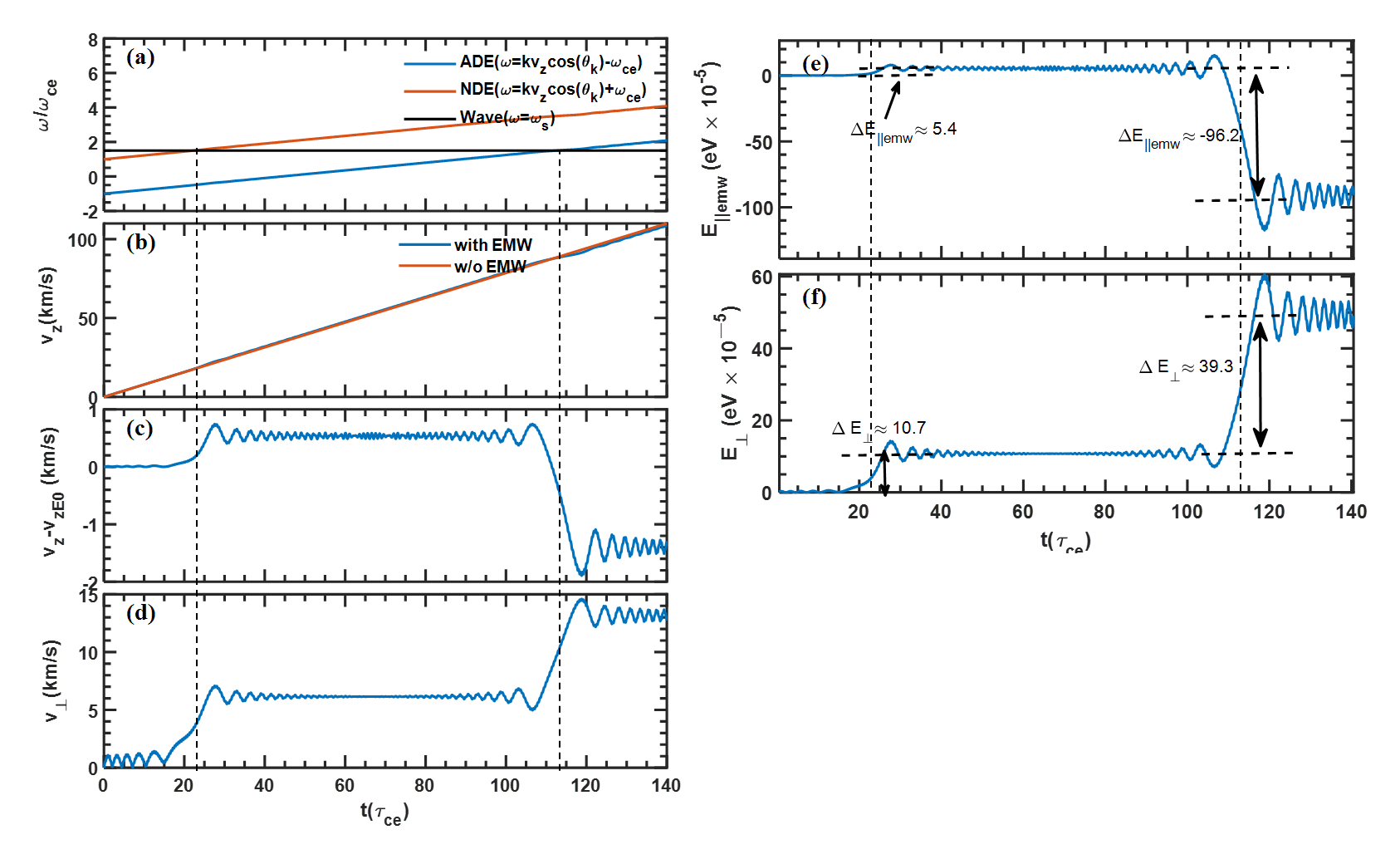


Figure . Kinetic evolution of electrons in a magnetic field with electromagnetic wave during acceleration. (a) frequencies of ADE, NDE, and source wave frequency. refers to the angle between k and z, here = 0. (b) The parallel velocity vz in the case with and without the electromagnetic wave. (c) The change of parallel velocity caused by the electromagnetic wave. (d) The cyclotron velocity .(e) The parallel kinetic energy transferred to electron by the electromagnetic wave. (f)The evolution of gyro-kinetic energy.

Section 2.2 : Validation of energy transfer ratio

As shown in fig. 5(a), around 23, the normal doppler frequency matches that of the induced wave, leading to a rapid increase in the cyclotron velocity (fig. 5 (b)). Simultaneously, the change in parallel velocity induced by the electromagnetic wave also increases. This phenomenon can be interpreted as the electron cyclotron system absorbing a photon during the Normal Doppler Effect, resulting in an increase in both parallel kinetic energy and cyclotron energy. The change in parallel kinetic energy caused by the electromagnetic wave is shown in fig. 5(e), where . The increase in cyclotron energy is shown in fig. 5(e), where . The enery transfer ratio between interanl energy and kinetic energy during resonance is given by . According to quantum theory, the energy ratio is given by eq.. Here m =1 for NDE and k = along z axis ,the resonant velocity vz 19 103m/s and . Finally, , which is in close agreement with the simulation results.

The Anomalous Doppler Effect begins to emerge when the time reaches 113 , where as shown in fig. 5(a). At this point, the parallel velocity begins to scatter into the perpendicular direction, evident from the decrease in and the increase in as seen in fig.5 (c) and fig.5 (d). During the resonant period, the changes in parallel kinetic and gyro-kinetic energies caused by electromagnetic wave are calculated as and . The enery transfer ratio is . According to quantum theory, the change ratio of =,where , and k = 105 /m, vz = 90 km/s. The quantum theory results are in good agreement with the numerical calculations. The energy change ratio is also derived in the Appendix, based on classical theory.

Section 2.2 : Varlidation of the relationship with wave angular momentum.



Figure . Velocity evolution caused by induced wave with linear, right-circular and left-circular polarization. (a) The cyclotron velocity .(b) The change of parallel velocity caused by the electromagnetic wave.

Fig.6(a-b) illustrate the velocity evolution under linear polarization of El, right-circular polarization (m = -1) and left-(m = 1).The work done on electron by the electromagnetic wave, Eemw, as depicted in fig. 6(c), consists of parallel direction, as previously described, and the gyro-kinetic energy . The latter is calculated as , where is determined from the electric and magnetic field forces, and represents the cyclotron velocity. All these parameters can be readily obtained from numerical results and integrated discretely.

The three types of polarization waves are investigated under the same scenario set as before. As a result, the right-hand circularly polarized wave (m = 1) caused a velocity change only at around 23τce, while the left-hand circularly polarized wave (m = -1) caused a velocity change only at around 113τce. This indicates that the right-circularly polarized wave is responsible for NDE, while the left-hand circularly polarized wave is responsible for ADE, which agrees well with the quantum analysis

The process can be understood as follows: For an electromagnetic wave with right-hand polarization propagating along the magnetic field, the electron in the magnetic field undergoes right-hand circular motion. When its parallel velocity satisfies the condition, known as the Normal Doppler Resonance condition, the electron, in its co-moving cyclotron frame, perceives the wave frequency as equal to its rotational frequency. Consequently, the electron resonates and absorbs the electromagnetic wave as indicted in fig. 6 (c) at 23. According to the conservation of angular momentum and parallel momentum, both the cyclotron velocity and parallel velocity increase, as the electromagnetic wave carries positive angular momentum (in the same direction as the cyclotron electron's angular momentum) and parallel momentum, which correspond to ℏ and ℏk in quantum physics.

For a left-circularly polarized electromagnetic wave, the resonance and scattering process occurs when the electron velocity satisfies the condition, known as the ADR. In the frame of the cyclotron electron, the electromagnetic wave has the same frequency and rotational direction with electron since the electron’s velocity exceeds the wave phase velocity. Since the electromagnetic wave performs negative work on the electron, as shown in fig. 6 (c) at 113, where Eemw is negative for a left-hand polarization wave, this is equivalent to the electron emitting an electromagnetic wave with the same properties as the induced wave. Because the emitted wave has left-hand circular polarization and positive momentum—corresponding to −ℏ and ℏk in quantum physics—the cyclotron velocity increases while the parallel velocity decreases, to keep the conservation of angular momentum and momentum.

For left-circularly polarized wave, where the angle momentum m = -1 , resonance occurs only at , as shown in fig.7 . This behavior differs from previous results, where resonance for plane E.M wave could occur at any integer m satisfying ,as shown in eq.(36) and eq.(37) of the paper[[40](#_ENREF_40)], but agree well with angular momentum conservation analysis.

图表

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Figure .(a) Frequency of w=under different m values and induced wave frequencies. (b) Perpendicular velocity evolution under the left-circularly polarized wave, where m = -1.

Section 3 : Discussion

Based on the momentum and angular momentum conservation analysis , we analyze the case where k is oriented opposite to v||. In this case, if a cyclotron electron emits a photon with left-hand circular polarization and momentum , where the angular momentum carried by photon is , then after the emission, the change in internal energy is ΔU = and the change in translational kinetic energy ΔT =. However , if the emitting photon have right-circular polarization and momentum , the change of internal energy becomes ΔU = ,while the translational kinetic energy still is ΔT =. This would violate the conservation of energy, as it is not possible for an electron to emit a photon while simultaneously increasing its total energy. Consequently, for a plane electromagnetic wave, only the left-circularly polarized component can resonate with an electron moving opposite to v||.

Section 4 : Conclusion

This paper presents simple yet useful method to analysis the resonant process of NDE and ADE. The quantum method, combined with an angular momentum conservation analysis, illustrates that the parameter m in the resonant condition is directly related to the angular momentum of the resonant wave. Numerical simulations based on the VPA method are also provided, confirming the correctness of the quantum results regarding both the angular momentum relationship between m and the energy transfer ratio.

Appendix

Classical analysis of Anomalous Doppler Resonant, here we would like give a brief derivation of energy transformation through classical dynamic equation:

Consider the same senario as used in simulation but without static electric field,ignore the relativistic effect , the equation of motion of electron is given by

Consider ,where is the unit vector of wave vector of E.M wave, which along withz axis , use and dot on both side of eq. and eq., substitute and simplify the equations, we have

Here , the total energy change of electron can be expressed as:

The sign of determine whether the electromagnetic (E.M.) wave undergoes “emission” ()or “absorption” () E.M wave, and this is dependent on the phase difference between and .

From eq. we have

Substitute eq. into eq. , we have

Integrate eq. ,we have

Here C0 refers to the initial value. The change in velocity is constrained to a circular trajectory, as shown in fig.8. At the normal Doppler resonance (NDR), where , an increase in increase corresponds to an increase in . In contrast, at the anomalous Doppler resonance (ADR), where , an increase in corresponds to a decrease in .

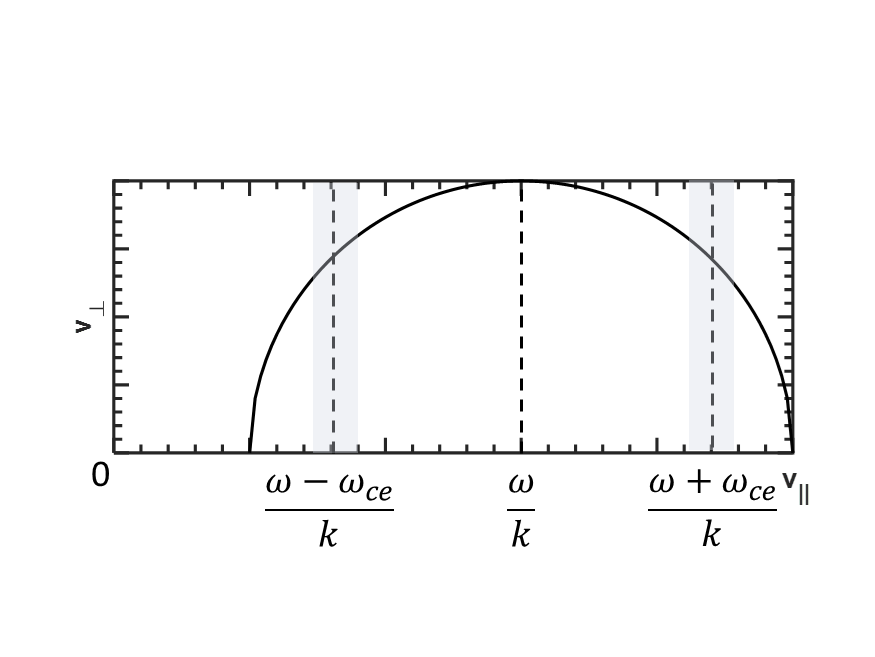


Figure . The trajectory curve of ()

The change of energy in translational energy and gyro-kinetic energy can be written as fellow

From eq.,we have

Combined eq. and eq.,we have

According to resonant condition , substituting with in eq. , we obtain:

Which agree with quantum result as eq..

Reference

[1] Tamm I E 1959 General characteristics of radiation emitted by systems moving with superlight velocities with some applications to plasma physics *Nobel Lectures* **18** 122-33

[2] Frank I 1960 Optics of Light Sources Moving in Refractive Media: Vavilov-Cherenkov radiation, though interesting, is but an experimental instance of a more general problem *Science* **131** 702-12

[3] Ginzburg V L 1960 Certain theoretical aspects of radiation due to superluminal motion in a medium *Soviet Physics Uspekhi* **2** 874

[4] Shustin E, POPOVICH P and Kharchenko I 1971 Transformation of Electron Beam Distribution Function Following Cyclotron Interaction with a Plasma *SOVIET PHYSICS JETP* **32**

[5] Ginzburg V and Frank I 1946 Radiation from a uniformly moving electron passing from one medium to another *Journ. of Experimental and Theoretical Physics (JETP) V* **16** 15-26

[6] Nezlin M V 1976 Negative-energy waves and the anomalous Doppler effect *Soviet Physics Uspekhi* **19** 946

[7] Santini F, Barbato E, De Marco F, Podda S and Tuccillo A 1984 Anomalous Doppler resonance of relativistic electrons with lower hybrid waves launched in the Frascati tokamak *Physical review letters* **52** 1300

[8] Kho T and Lin A 1988 Slow-wave electron cyclotron maser *Physical Review A* **38** 2883

[9] Liu J, Wang Y and Qin H 2016 Collisionless pitch-angle scattering of runaway electrons *Nuclear Fusion* **56** 064002

[10] Wang Y, Qin H and Liu J 2016 Multi-scale full-orbit analysis on phase-space behavior of runaway electrons in tokamak fields with synchrotron radiation *Physics of Plasmas* **23**

[11] Guo Z, McDevitt C J and Tang X-Z 2018 Control of runaway electron energy using externally injected whistler waves *Physics of Plasmas* **25**

[12] Liu C, Hirvijoki E, Fu G-Y, Brennan D P, Bhattacharjee A and Paz-Soldan C 2018 Role of kinetic instability in runaway-electron avalanches and elevated critical electric fields *Physical review letters* **120** 265001

[13] Shi X, Lin X, Kaminer I, Gao F, Yang Z, Joannopoulos J D, Soljačić M and Zhang B 2018 Superlight inverse Doppler effect *Nature Physics* **14** 1001-5

[14] Filatov L and Melnikov V 2021 The Role of the Anomalous Doppler Effect in the Interaction of Energetic Electrons with Whistler Turbulence in Flare Loops *Geomagnetism and Aeronomy* **61** 1183-8

[15] Artsimovich L, Bobrovskii G, Mirnov S, Razumova K and Strelkov V 1967 Thermal insulation of plasma in the “Tokamaks” *Soviet Atomic Energy* **22** 325-31

[16] Kadomtsev B and Pogutse O 1968 Electric conductivity of a plasma in a strong magnetic field *Sov. Phys. JETP* **26** 1146

[17] Spong D A, Heidbrink W, Paz-Soldan C, Du X, Thome K, Van Zeeland M, Collins C, Lvovskiy A, Moyer R and Austin M 2018 First direct observation of runaway-electron-driven whistler waves in tokamaks *Physical Review Letters* **120** 155002

[18] Liu Y, Zhou T F, Hu Y M, Zhao H L, Zhu Z Y, Liu X, Ling B L, Zhou R J and Zhang T 2019 Intense intermittent radiation at the plasma frequency on EAST *Epj Web Conf* **203**

[19] Gorozhanin D V, Ivanov B I, Khoruzhiy V M, Onishchenko I N and Miroshnichenko V I 1997 Waves excitation at anomalous Doppler effect for various electron beam energies *Icpp 96 Contributed Papers - Proceedings of the 1996 International Conference on Plasma Physics, Vols 1 and 2* 402-5

[20] Chen Z Y, Wan B N, Ling B L, Gao X, Du Q, Ti A, Lin S Y, Sajjad S and Team H- 2007 Runaway electron beam instability in slide-away discharges in the HT-7 tokamak *Chinese Physics Letters* **24** 3195-8

[21] Castejon F and Eguilior S 2003 Particle Dynamics under Quasi-linear Interaction with Electromagnetic Waves. Centro de Investigaciones Energeticas)

[22] Benford J, Swegle J A and Schamiloglu E 2007 *High power microwaves*: CRC press)

[23] Zhang Q, Zhang Y, Tang Q and Tang X-Z 2024 Self-mediation of runaway electrons via self-excited wave-wave and wave-particle interactions *arXiv preprint arXiv:2409.15830*

[24] Ginzburg N 1979 Nonlinear theory of electromagnetic wave generation and amplification based on the anomolous Doppler effect *Radiophysics and Quantum Electronics* **22** 323-30

[25] Ginzburg V L 2005 Radiation from uniformly moving sources (Vavilov-Cherenkov effect, transition radiation, and some other phenomena) *Acoust Phys+* **51** 11-23

[26] Coppi B, Pegoraro F, Pozzoli R and Rewoldt G 1976 Slide-Away Distributions and Relevant Collective Modes in High-Temperature Plasmas *Nuclear Fusion* **16** 309-28

[27] Frolov V P and Ginzburg V L 1986 Excitation and Radiation of an Accelerated Detector and Anomalous Doppler-Effect *Physics Letters A* **116** 423-6

[28] Ginzburg V L 1996 Radiation by uniformly moving sources (Vavilov–Cherenkov effect, transition radiation, and other phenomena) *Physics-Uspekhi* **39** 973

[29] Arnaut H and Barbosa G 2000 Orbital and intrinsic angular momentum of single photons and entangled pairs of photons generated by parametric down-conversion *Physical review letters* **85** 286

[30] Liu H, He X, Chen S and Zhang W 2004 Particle acceleration through the resonance of high magnetic field and high frequency electromagnetic wave *arXiv preprint physics/0411183*

[31] Qian B L 1999 An exact solution of the relativistic equation of motion of a charged particle driven by a circularly polarized electromagnetic wave and a constant magnetic field *Ieee T Plasma Sci* **27** 1578-81

[32] Weyssow B 1990 Motion of a single charged particle in electromagnetic fields with cyclotron resonances *Journal of plasma physics* **43** 119-39

[33] Gogoberidze G and Machabeli G 2005 On the origin of the circular polarization in radio pulsars *Monthly Notices of the Royal Astronomical Society* **364** 1363-6

[34] Roberts C S and Buchsbaum S 1964 Motion of a charged particle in a constant magnetic field and a transverse electromagnetic wave propagating along the field *Physical Review* **135** A381

[35] Bourdier A and Gond S 2000 Dynamics of a charged particle in a circularly polarized traveling electromagnetic wave *Physical Review E* **62** 4189-206

[36] Nusinovich G S, Korol M and Jerby E 1999 Theory of the anomalous Doppler cyclotron-resonance-maser amplifier with tapered parameters *Physical Review E* **59** 2311

[37] Nusinovich G S, Latham P and Dumbrajs O 1995 Theory of relativistic cyclotron masers *Physical Review E* **52** 998

[38] Qian B L 2000 Relativistic motion of a charged particle in a superposition of circularly polarized plane electromagnetic waves and a uniform magnetic field *Physics of Plasmas* **7** 537-43

[39] Zhang R, Liu J, Qin H, Wang Y, He Y and Sun Y 2015 Volume-preserving algorithm for secular relativistic dynamics of charged particles *Physics of Plasmas* **22**

[40] Dendy R O 1987 Classical single-particle dynamics of the anomalous Doppler resonance *Physics of Fluids* 30