Analysis Anomalous Doppler Effect from quantum theory to classical dynamic simulation

Abstract:

A quantum model combined with angular momentum conservation is established to analyze the process of Normal doppler effect and Anomalous Doppler Effect, illustrating that the resonance process is related to the angular momentum of the wave. The angular momentum resonant condition is numerically tested, and the energy change ratio between parallel and gyrokinetic energies during electron–wave resonance is calculated, showing strong agreement with quantum theory.

I. Introduction:

The Anomalous Doppler Effect (ADE)[[1-4](#_ENREF_1)], in which the observed frequency shift behaves contrary to the conventional Doppler Effect under specific conditions, was first theoretically predicted by Soviet physicist Vitaly L. Ginzburg[[5](#_ENREF_5)]. This phenomenon occurs when a moving system’s velocity exceeds the phase velocity of light in the medium, it transfers its kinetic energy to its internal energy while emitting radiation. A notable example, discussed by Frank in his 1958 Nobel lecture[[2](#_ENREF_2)], demonstrates that radiation emission does not result from atomic transitions from a higher (excited) state to a lower state, as is typical, but rather occurs inversely—from a lower state to a higher state—where the energy is supplied by the system’s translational kinetic energy. This intriguing theoretical prediction has attracted significant attention and has motivated extensive research[[6-14](#_ENREF_6)].

In 1967, Artsimovich[[15](#_ENREF_15)] observed discrepancies in tokamak experiments: measurements of electron temperature derived from diamagnetic signals stronger than derived from electrical conductivity measurement. This anomaly, unrecognized at the time, may represent the first experimental observation of ADE. It was not until 1968 that B. B. Kadomtsev[[16](#_ENREF_16)]  identified the that cause as ADE, wherein electron’s longitudinal velocity scatter to transverse velocity under resonant ADE conditions. This process amplifies the diamagnetic effect beyond contributions from thermal motion alone. After that, more phenomena related to ADE are observed, such as electron beam scattering in magnetic field vacuum tube[[4](#_ENREF_4)], wave radiation [[17-19](#_ENREF_17)] and runaway electron instability in tokamaks [[20](#_ENREF_20), [21](#_ENREF_21)]. Applications based on ADE have also emerged in various areas, such as high-power microwave generation [[22](#_ENREF_22)] and runaway electron suppression in tokamaks[[11](#_ENREF_11), [23](#_ENREF_23)].

The physics of the Anomalous Doppler Effect (ADE) was previously explained based on the quantum analysis provided by Frank and Ginzburg[[2](#_ENREF_2), [24](#_ENREF_24)]. In this paper, building on Ginzburg’s quantum analysis and incorporating angular momentum conservation, we present a more detailed analysis of ADE, offering further insights into the relationship between ADE and wave angular momentum. Despite the simplicity of the model, to the best of our knowledge, the analysis of angular momentum conservation during the ADE process has not been presented or mentioned before.

Additionally, a numerical simulation of a single particle resonating with an electromagnetic (EM) wave in the presence of uniform static electric and magnetic fields is conducted based on classical dynamical equations. This simulation demonstrates the relationship between the wave’s angular momentum and the resonance condition. The energy transfer ratio from the electron's kinetic energy to its gyrokinetic energy during the resonance with the EM wave is computed numerically, and the results show strong agreement with the formula derived from quantum analysis.

The remainder of this paper is organized as follows. Section II presents the quantum analysis based on angular momentum conservation. Section III describes the numerical setup and methodology, including illustrations of the temporal evolution of velocity and kinetic energy. Section IV discusses the energy transfer ratio and polarization characteristics. Finally, Section V provides a brief discussion and conclusion.

II. Quantum analysis of ADE

We begin with the quantum theory proposed by V.L. Ginzburg[[25](#_ENREF_25)] based on the energy-momentum conservation, and then incorporate angular momentum conservation to illustrate the relationship of wave’s angular momentum and Landau level n under the resonant condition , where is wave’s angular frequency , represents the wave vector, means the velocity of electron and is the electron cyclotron angular frequency in static magnetic field, n = is the quantum number of the Landau levels[[26](#_ENREF_26)].

When a charged particle moves through a medium at a speed greater than the phase velocity of light in that medium, it induces polarization in the surrounding molecules. As these molecules return to their equilibrium state, they emit electromagnetic radiation. The constructive interference of these emissions produces the characteristic Cherenkov radiation, forming a cone-shaped wavefront as shown in fig.1. The direction of Cherenkov radiation is constrained to the Cherenkov radiation angle ,where c′ is the speed of light in the medium and v is the velocity of the charged particles.

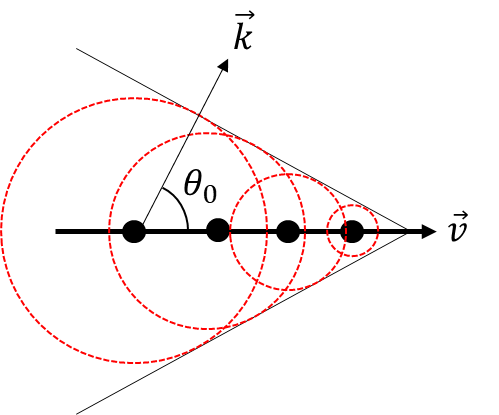


Figure . Schematic diagram of Cherenkov Radiation. The black points stand for the snapshot of the electron at different times, the read dash circle refers to the current radiation surface from the previous electron.

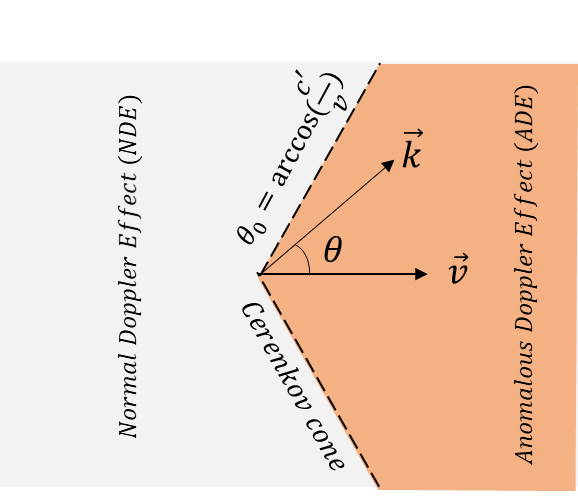


Figure .The region of Anomalous Doppler Effect (ADE) and Normal Doppler Effect (NDE).

However, when the electron is replaced by a system possessing internal energy—such as an oscillator or a cyclotron electron in a magnetic field—the direction of the emitted photon is no longer determined by the interference of secondary waves and can instead occur in any direction. Considering a scenario where the system emits a photon with angular frequency 𝜔 and wavevector k, the emission process must satisfy both energy and momentum conservation:

Here the and represent the kinetic energy and internal energy of the system while subscripts of 1 and 2 refer to before and after emitting a photon. p represents the momentum of the system and ℏ represnts reduced Planck's constant. Assumpting that photon’s energy is far less than the initial kinetic energy , the losses of kinetic energy after emitting a photon can be expressed as , where v is the velocity of the system before emitting a photon and **.** Thus, the change of internal energy can be expressed as

Here, . When the system's velocity exceeds the speed of light in the medium , the sign of allows the radiation to be categorized into three distinct regions, as illustrated in fig. 2.

* For , . The system produces photons by consuming its own internal and kinetic energy, this region refers to the Normal Doppler Effect (NDE).
* For , , the loss of kinetic energy by the system is completely converted into photon energy; this line refers to the Cerenkov Effect.
* For , , this region is referred to the Anomalous Doppler Effect (ADE), where the system gains internal energy after emitting photons. It means the loss of kinetic energy is converted to photons and the system’s internal energy.

When the system velocity exceeds the speed of light (v < c’), all three effects are possible while the system velocity is less than the speed of light (v > c’), only Normal Doppler Effect exists.

In previous paper, the change of internal energy is given as where m represents the Landau level ,as given by V.L. Ginzburg[[25](#_ENREF_25)] ,Coppi, [[26](#_ENREF_26)],Frolov[[27](#_ENREF_27)] ,Frank[[2](#_ENREF_2)],Tamm[[1](#_ENREF_1)] and Nezlin[[6](#_ENREF_6)] . The above content revisits the foundational work of V.L. Ginzburg. In the present paper, it is further demonstrated that m also represents the quantum number associated with the angular momentum of the emitted photon.

Let’s consider the process in which an electron cyclotron system under a uniform magnetic field emits a photon, as shown in fig.3. The moving electron has the velocity vz along the background magnetic field and the cyclotron velocity. The kinetic energy along z is , where refers to the Lorentz factor. The internal energy represents as . Assume the angular moment of the system before and after emitting a photon is L1 ,and L2, respectively. The angular moment of photon is n. According to the angular momentum conservation, we have

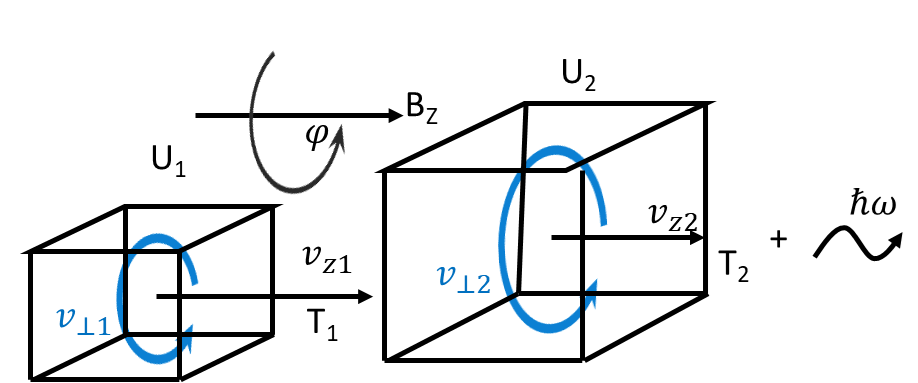


Figure . Schematic diagram of electron cyclotron system before and after emitting a photon. Here U2>U1, T2<T1.

Since the magnetic field is aligned along z direction, the angular momentum of electron cyclotron along z is represented as Lz. According to the quantum theory, the electron wave in the static magnetic field can be expressed as:

With the term represents as normalized coefficient, A is the vector potential and s is the position. For cyclotron-electron in magnetic field, s = rφ , where r refers to cyclotron radius and φ refers to cyclotron angle. The z component of the orbital angular momentum operator can be expressed in spherical coordinates as

Combining eq. () with eq. (), we have

As a result, the eigenvalue of can be expressed as

With ,and , the eq. () is presented as

Here is the electron rest mass, is the Lorentz factor and is the electron cyclotron frequency in rest frame (). The angular momentum conservation in z direction is The variation in the angular momentum of the electron along z is presented as

With m is the number of photon’s angular momentum in z direction.The internal energy change is given by , with the eq. (), will be transformed as :

According to the eq.\* and eq.\*, the change in electron energy could be presented as

Here, represents the loss of kinetic energy , represents the energy of the photon, and represents the change in the electron cyclotron energy (internal energy change). The change ratio of internal energy and kinetic energy can be expressed as

This results is a critical criterion to compare with the classical dynamic simulation in the section 3 .

After simpifying the eq.\*, we finally have the classical wave-particle resonant condition

Based on the analysis above, the m actually represents the quantum number of angular momentum of the photon. There are three scenarios about the sign of m.

* For , , the cyclotron electron internal energy decreases after emitting a photon, and the emitted photon will have right-hand circular polarization if angular momentum quantum number m = 1. This process is called the Normal Doppler Effect .
* For , , the Cherenkov Effect occurs, where the emitted photon does not cause any change in the internal energy of the cyclotron electron.
* For , , the Anomalous Doppler Effect (ADE) occurs,resulting in an increase in the internal energy of the cyclotron electron and the emission photo will have left-hand circular polarization if the angular momentum quantum number m = -1.

The aforementioned analysis is based on spontaneous emission. However, similar to laser emission, this conservation model is also applicable to stimulated emission, wherein the emitted photon is generated with the same frequency, direction, and phase as the incident photon. External electromagnetic waves can serve as resonant fields to trigger cyclotron electrons in a magnetic field to emit or absorb waves, providing a framework for analyzing the Anomalous Doppler Effect. For a external electromagnetic wave as plane wave, the wave angular moment number can be devided into . While for , it indicates that the resonant wave possesses a helicon structure. In this study, we consider only the primary resonant conditions : the ADE resonantce condition, **,**

and the NDE resonance condition, .

Section 2 : Classical dynamic simulation of ADE

The ADE process has been analyzed based on quantum theory, demonstrating that the angular momentum of the stimulated electromagnetic wave determines the resonance condition. Specifically, only angular momentum with m < 0 corresponds to the ADE process, while m >0 corresponds to the NDE process. The enery transfer ratio between interanl energy and kinetic energy during resonance can be expressed as , and the ratio between the energy done by photon and the change of kinetic energy during resoance can be expressed as .

Section 2.1 : Numerical simulation setup

To analyze the ADE process from the perspective of classical dynamics and to provide a direct comparison between quantum and classical dynamic results, the following scenario is established: The uniform magnetic field is set along the z-direction. The electrostatic field , which on the opposite direction to as shown in Fig. \*, is used to accelerate the electron. A plane, linearly polarized slow electromagnetic wave is established as induced wave, characterized by frequency 𝜔 and wavevector **k**. This type of slow wave commonly exists in plasmas or corrugated waveguides. \*\*\*

The six-dimensional phase space of an electron, described by its position **r** and momentum **p,** is presented in eq.\*. The vectors **E** and **B** represent the total field, including both static and electromagnetic components. Here, c denotes the speed of light in vacuum, e represents to the electron’s charge and m0 is the electron’s mass in rest frame.

To simulate the evolution of r and p , the eq.\* is discreted as the form of eq.\* based on the Volume-Preserving Algorithm\*\*\*. Here the k is the iteration step and the operator Cay(A) denotes the Cayley transform of matrix A [\*].

The dimensionless parameters are momentum , magnetic field total electric field ,time step , and position respectively, where the is the electron cyclotron period and is Lorentz factor. The dimensionless magnetic matrix B\* is writen as eq.(\*)

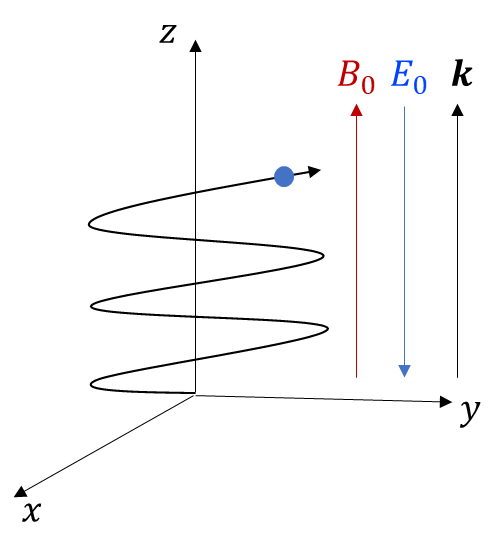


Figure .The uniform static magnetic field is set along the z axis, the electrostatic field E0 is oriented opposite to the B0 field, and the wavevector k is aligned parallel to the B0 field.

To illustrate the system evolution and achieve highly efficient calculation, the parameters are set as following: background magnetic field , wave angular frequency where , wavevector , the electric field component of the electromagnetic wave , and the electrostatic field is .The induced wave with linear polarization can be expressed as E = Ew cos( where r is the position of electron in the frame. The time resolution is always chosen to satisfy 50()) to ensure the accuracy of the simulation.

The evolution of the electron’s motion is shown in Fig. 5. As the electron accelerates in the electrostatic field (Fig. 5(b)), the resonant frequencies increase simultaneously (Fig. 5(a)). The change of parallel velocity caused by electromagnetic wave can be quantified as as shown in Fig.5(c), where vz represents the parallel velocity under the given scenario, while vzE0​ denotes the parallel velocity resulting solely from the electrostatic field, which can be calculated using a theoretical equation. The cyclotron velocity is shown in Fig.5 (d). The work done by electromagnetic wave is shown in Fig.5 (e), which can be calculated by integrating the power with time as ,and . Since all discrete date points are available from the simulation, it is no difficult to integrate all the discreate date over time. Figure.5(f) shows the cyclotron energy evolution with time, where .

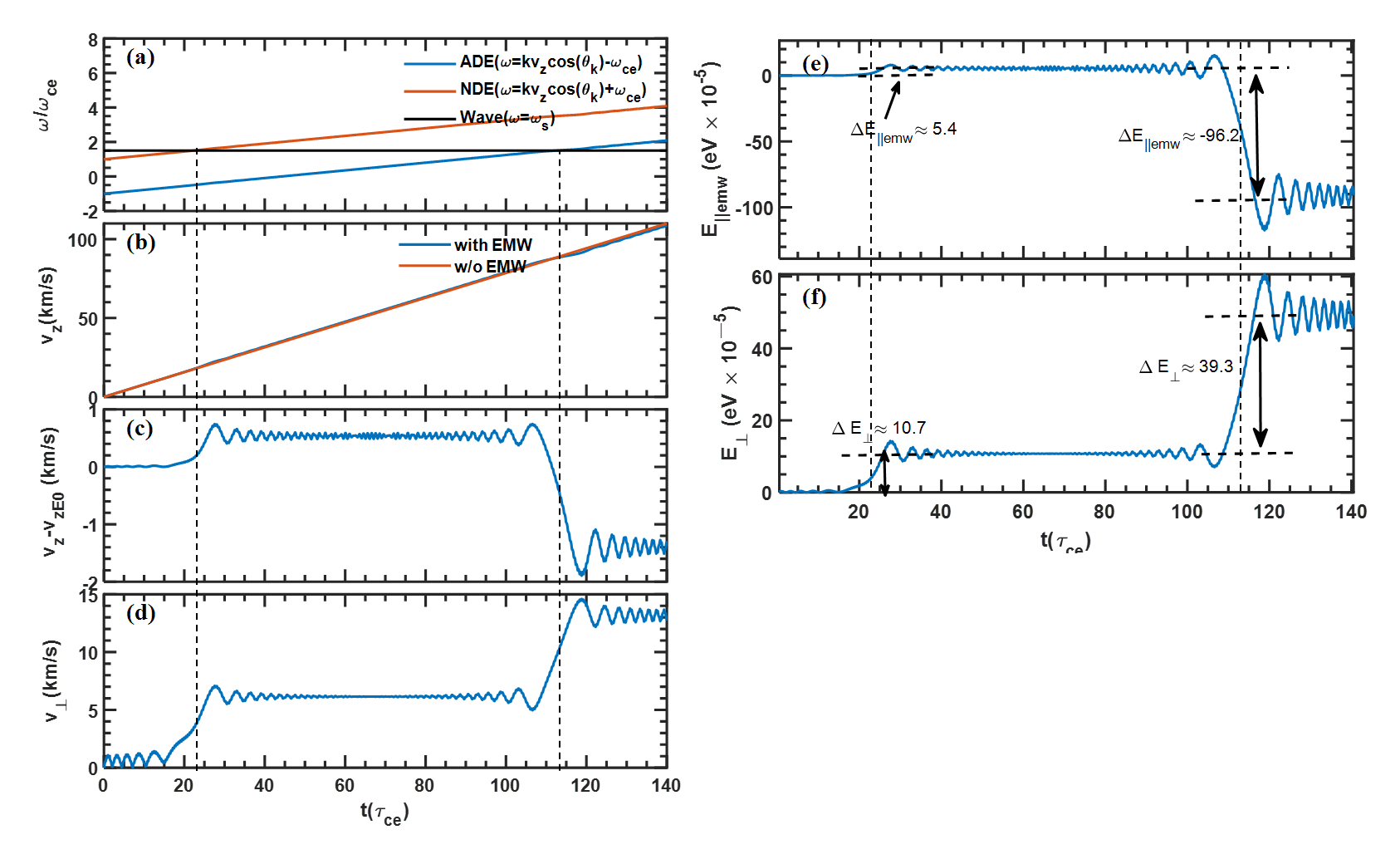


Figure .Kinetic evolution of electrons in a magnetic field with electromagnetic wave during acceleration. (a) Wave frequencies of Anomalous Doppler Effect (ADE), Normal Doppler Effect (NDE), and source wave frequency. refers to the angle between k and z, here = 0. (b) The parallel velocity vz in the case with and without the electromagnetic wave. (c) The change of parallel velocity caused by the electromagnetic wave. (d) The cyclotron velocity .(e) The parallel kinetic energy by electromagnetic wave. (f)The evolution of cyclotron energy.

Section 2.2 : Varlidation of energy tranfer ratio

At around 23 , the Normal Doppler Frequency matches that of the induced wave (Fig.5(a)), leading to a rapid increase in the cyclotron velocity (Fig.5 (b)). Simultaneously, the change in parallel velocity induced by the electromagnetic wave also increases. This phenomenon can be interpreted as the electron cyclotron system absorbing a photon during the Normal Doppler Effect, resulting in an increase in both parallel kinetic energy and cyclotron energy. The change in parallel kinetic energy caused by the electromagnetic wave is shown in Fig. 5(e), where . The increase in cyclotron energy is shown in Fig.5 (e), where . The enery transfer ratio between interanl energy and kinetic energy during resonance is given by . According to quantum theory, the energy ratio is given by

Here m =1 for NDE and k = along z axis ,the resonant velocity vz 19 103m/s and . Finally, , which is in close agreement with the simulation results.

The Anomalous Doppler Effect begins to emerge when the time reaches 113 , where as shown in Fig.5 (a). At this point, the parallel velocity begins to scatter into the cyclotron direction, evident from the decrease in and the increase in as seen in Fig.5 (c) and Fig.5 (d). During the resonant period, the changes in parallel and cyclotron energies caused by electromagnetic wave are calculated as and . The enery transfer ratio is . According to quantum theory, the change ratio of =,where , and k = 105 /m, vz = 90 km/s. The quantum theory results are in good agreement with the numerical calculations.

Section 2.2 : Varlidation of the relationship with wave polarization

The induced linear polarized wave can be sperated as the combined of right- hand polarization and left-hand polarization wave

Where the right-hand polarization wave is



Figure . Velocity evolution caused by induced wave with linear, right-hand and left-hand polarization

And the left-polarization wave is

By subjecting the electron to three types of polarized waves E , and , the results are shown in Fig. 6. The work done on electron by the electromagnetic wave, Eemw, as depicted in Fig. 6(c), consists of the work done in the parallel direction, as previously described, and the work done in the cyclotron direction . The latter is calculated as , where is determined from the electric and magnetic field forces, and represents the cyclotron velocity. All these parameters can be readily obtained from numerical results and integrated discretely.

The three types of polarization waves are investigated under the same scenario set as before, and the velocity evolution is demonstrated in Fig. 6. As a result, the right-hand circularly polarized wave caused a velocity change only at around 23τce, while the left-hand circularly polarized wave caused a velocity change only at around 113τce. This indicates that the right-hand circularly polarized wave is responsible for NDE, while the left-hand circularly polarized wave is responsible for ADE, which agrees well with the quantum analysis.

The process can be understood as follows: For an electromagnetic wave with right-hand polarization propagating along the magnetic field, the electron in the magnetic field undergoes right-handed circular motion. When its parallel velocity satisfies the condition, known as the Normal Doppler Effect (NDE) resonance condition, the electron, in its co-moving cyclotron frame, perceives the wave frequency as equal to its rotational frequency. Consequently, the electron resonates and absorbs the electromagnetic wave as indicted in Fig.6(c) at 23, where Eemw is positive for right-hand polarization wave. According to the conservation of angular momentum and parallel momentum, both the cyclotron velocity and parallel velocity increase, as the electromagnetic wave carries positive angular momentum and parallel momentum, which correspond to ℏ and ℏk in quantum physics.

For a left-hand polarized electromagnetic wave, the resonance and scattering process occurs when the electron velocity satisfies the condition, known as the Anomalous Doppler Effect (ADE) resonance condition. In the reference frame of the cyclotron electron, the electromagnetic wave has the same frequency and rotational direction as the electron’s velocity exceeds the wave phase velocity. This leads to a change in the perceived rotational direction of the wave in the electron’s frame. Since the electromagnetic wave performs negative work on the electron, as shown in Fig.6(c) at 113, where Eemw is negative for left-hand polarization wave, this is equivalent to the electron emitting an electromagnetic wave with the same properties as the induced wave. Because the emitted wave has left-hand circular polarization and positive momentum—corresponding to −ℏ and ℏk in quantum physics—the cyclotron velocity increases while the parallel velocity decreases, to keep the conservation of angular momentum and momentum. This process is consistent with the scattering phenomenon. An interesting phenomenon observed here is that the negative power for linear polarization is greater than that for left-hand polarization at 113. This occurs because, under linear polarization, the cyclotron electron system gains more cyclotron energy during the NDE resonance, allowing it to store more energy, which is subsequently released more emission during the ADE process.

Section 3 : Discussion

This study provides a new perspective on electron heating and current drive by electromagnetic waves. For instance, during the NDE process for a plane wave with right-hand polarization, given a certain wave energy input into the plasma, the electron heating coefficient can be evaluated as where m=1 and , Meanwhile, the current drive coefficient can be expressed as . Both heating and current drive occur only at the resonant velocity vz=(kz . Therefore, to achieve efficient heating, in addition to considering the resonant velocity, the ratio should also be regarded as a crucial factor. The heating effciency is limited due to only and (the angle between k and z) is determined by operation system while k and are dictated by plasma environment. However, by utilizing helicon wave, , and m>1 become adjustable parameters, which could potentially expand the operational range and enhance the heating efficiency. On the other hand, the ADE process induces electron velocity scattering, which presents a potential method for suppressing runaway electrons in tokamaks. Actually, the heating process and current driven or scattering process is a nonlinear effect, for example , the complex environment and spectral width , and cannot be treated by the analysis offered in this letter. Neverthless , although a strict comparsion is not appropriate , it may be heuristic to explore the complex phenomenon from single electron, and get basic physis of wave-partical interaction.

Section 4 : Conclusion

The NDE and ADE processes have been analysed using both quantum theory and numerical simulations, with results showing strong agreement. The energy tranfer ratio from external wave to cycltron electron can be readily calculated using eq.\* and eq.\* ,this results may be worthy of exploration in heating , current driven and runaway suppression.

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